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Super Stolarsky-3 Mean Labeling of Quadrilateral Snake Graphs

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Abstract

Let G = (V, E) be a graph with p vertices and q edges. Let f: V (G) → {1, 2, …, p + q} be an injective function. For a vertex labeling f, the induced edge labeling f* (e=uv) is defined by

$$f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rceil \text{ (or) } \left\lfloor \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rfloor$$

. Then f is called a Super Stolarsky-3 Mean labeling if f(V(G)∪{f(e) / e ϵ E(G)} = {1, 2, …, p + q}.

A graph which admits Super Stolarsky-3 Mean labeling is called Super Stolarsky-3 Mean graphs.

In this paper, we investigate Super Stolarsky-3 Mean labeling of Quadrilateral Snake graphs.

Keywords - Graph, Super Stolarsky-3 Mean labeling, Quadrilateral Snake graph, Double Quadrilateral Snake graph, Triple Quadrilateral Snake graph, Four Quadrilateral Snake graph.

1. INTRODUCTION

All graphs G= (V, E) with p vertices and q edges are finite, simple and undirected. For a detailed survey of graph labeling we refer Gallian (2017) **[1]** . For all other standard terminologies and notations we follow Harary**[2]**. S.S. Sandhya, E.Ebin Raja Merly and S.Kavitha introduced a new type of Labeling called "**Stolarsky-3 Mean**

Labeling of Graphs" in [4]. In this paper we prove that Double quadrilateral Snake, Triple Quadrilateral Snake, Four Quadrilateral Snake graphs are Super Stolarsky-3 Mean labeling of graphs. The following definitions and theorems are useful for our present investigation.

A **walk** in which all the vertices $u_1, u_2, \ldots, u_n$ are distinct is called a path. It is denoted by P_n. A **Quadrilateral snake $\boldsymbol{Q_n}$** is obtained from a path$u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining $v_i$andw_i. That is, every edge of a path is replaced by a cycle C_4. **Double Quadrilateral snake D($\boldsymbol{Q_n}$)** consists of two Quadrilateral snakes that have a common path. **Triple Quadrilateral snake T($\boldsymbol{Q_n}$)** consists of three Quadrilateral snakes that have a common path. **Four Quadrilateral snake F($\boldsymbol{Q_n}$)** consists of Four Quadrilateral snakes that have a common path.

Definition 1.1: Let G = (V,E) be a graph with p vertices and q edges. Let f:V(G)→{1,2,…,p+q} be an injective function. For a vertex labeling f, the induced edge labeling f* (e=uv) is defined by

f* (e) = $\left\lceil \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rceil$ (or) $\left\lfloor \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rfloor$. Then f is called a Super Stolarsky-3 Mean labeling if f(V(G) ∪{f(e) / e ∈ E(G)}= {1, 2, …, p + q}. A graph which admits Super Stolarsky-3 Mean labeling is called Super Stolarsky-3 Mean graphs.

Theorem 1.2 [5]: Quadrilateral Snake Q_n is Super Stolarsky-3 Mean graph (S.S. Sandhya , E.Ebin Raja Merly and S.Kavitha).

2. MAIN RESULTS

Theorem 2.1: Double Quadrilateral Snake D (Q_n) is Super Stolarsky-3 Mean graph.

Proof:

Let D(Q_n) be the Double Quadrilateral Snake graph.

Consider a path $u_1, u_2, \ldots, u_n$.

To Construct D (Q_n). Join $u_i and\ u_{i+1}$ to four new vertices v_i , w_i, x_i , y_i , 1≤ i≤n-1.

Define a function **f** : V(D(Q_n)) →{ 1,2,…., p+q} by

f(u_i) = 12i -7 , 1≤ $i \le n$.

f(v_i) = 12i -8 , 1≤ $i \le n$.

f(w_i) = 12i -4 , 1≤ $i \le n$.

f(x_i) = 12i -7 , 1≤ $i \le n$.

f(y_i)= 12i -1 , 1≤ $i \le n$.

Then the edges are labeled with

f$(u_i u_{i+1})$ =12i -5, 1≤ $i \leq n-1$.

f$(u_i v_i)$ =12 i -10, 1≤ $i \leq n-1$.

f$(v_i w_i)$ =12 i -6, 1≤ $i \leq n-1$.

f$(w_i u_{i+1})$ =12i -2, 1≤ $i \leq n-1$.

f$(u_i x_i)$ =12i-9, 1≤ $i \leq n-1$.

f$(x_i y_i)$ =12i-3, 1≤ $i \leq n-1$.

f$(y_i u_{i+1})$ =12i, 1≤ $i \leq n-1$.

Then we get distinct edge labels.

Hence D (Q_n) is Super Stolarsky-3 Mean graph.

Example 2.2: The SuperStolarsky-3 Mean labeling of D(Q_4) is given below.

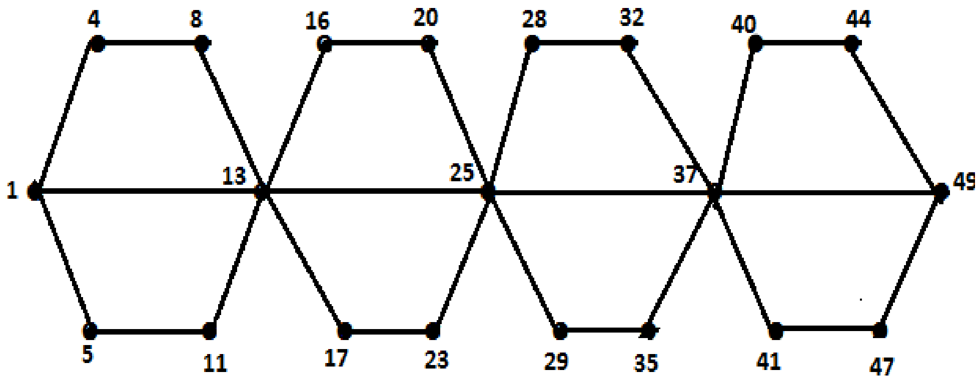

Figure 1

Theorem 2.3: Triple Quadrilateral Snake T(Q_n) is Super Stolarsky-3 Mean graph.

Proof:

Let T (Q_n) be the Triple Quadrilateral Snake graph.

Let P_n be the path $u_1, u_2, \dots, u_n$.

To Construct T(Q_n). Join u_i *and* u_{i+1} to six new vertices v_i , w_i, v_i' , w_i' *and* x_i , y_i , 1≤ i≤ n-1.

Define a function **f** : V(T(Q_n)) →{ 1,2,…., p+q} by

f(u_i) = 17i -16 , 1≤ $i \leq n$.

f(v_i) = 17i -13 , 1≤ $i \leq n$.

f(w_i) = 17i -6 , 1≤ $i \leq n$.

$\mathbf{f}(v_{i'})$ = 17i -11 , $1 \leq i \leq n$.

$\mathbf{f}(w_{i'})$ = 17i -5 , $1 \leq i \leq n$.

$\mathbf{f}(x_i)$= 17i -9 , $1 \leq i \leq n$.

$\mathbf{f}(y_i)$= 17 i -1 , $1 \leq i \leq n$.

Then the edges are labeled with

$\mathbf{f}(u_i u_{i+1})$ =17i -7, $1 \leq i \leq n-1$.

$\mathbf{f}(u_i v_i)$ =17 i -15, $1 \leq i \leq n-1$.

$\mathbf{f}(u_i v_{i'})$ =17 i -14, $1 \leq i \leq n-1$.

$\mathbf{f}(v_i w_i)$ =17i -10, $1 \leq i \leq n-1$.

$\mathbf{f}(v_{i'} w_{i'})$ =17i-8, $1 \leq i \leq n-1$.

$\mathbf{f}(w_i u_{i+1})$ =17i -3, $1 \leq i \leq n-1$.

$\mathbf{f}(w_{i'} u_{i+1})$ =17i -2, $1 \leq i \leq n-1$.

$\mathbf{f}(u_i x_i)$ =17i-12, $1 \leq i \leq n-1$.

$\mathbf{f}(x_i y_i)$ =17i-4, $1 \leq i \leq n-1$.

$\mathbf{f}(y_i u_{i+1})$ =17i, $1 \leq i \leq n-1$.

Then we get distinct edge labels.

Hence T (Q_n) is Super Stolarsky-3 Mean graph.

Example 2.4: The SuperStolarsky-3 Mean labeling of T(Q_4) is given below.

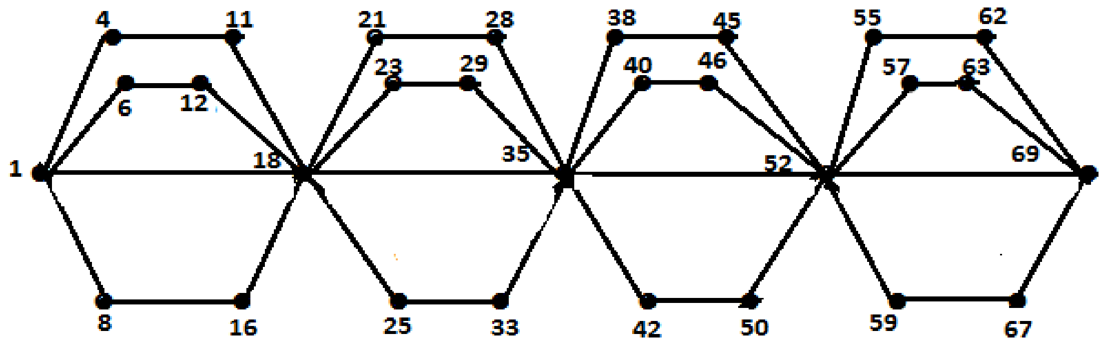

Figure 2

Theorem 2.5: Four Quadrilateral Snake F(Q_n) is Super Stolarsky-3 Mean graph.

Proof:

Let F(Q_n) be the Four Quadrilateral Snake graph.

Let P_n be the path$u_1, u_2, \ldots, u_n$.

To construct F(Q_n), Join $u_i and\ u_{i+1}$ to eight new vertices v_i , w_i, v_i' , w_i' , x_i , y_i and $x_{i'}$, $y_{i'}$, 1≤ i≤ n-1.

Define a function **f** : V(F(Q_n)) →{ 1,2,…., p+q} by

$\mathbf{f}(u_i)$ = 22i -21 , 1≤ $i \leq n$.

$\mathbf{f}(v_i)$ = 22i -18 , 1≤ $i \leq n$.

$\mathbf{f}(w_i)$ = 22i -10 , 1≤ $i \leq n$.

$\mathbf{f}(v_{i'})$ = 22i -16 , 1≤ $i \leq n$.

$\mathbf{f}(w_{i'})$ = 22i -7 , 1≤ $i \leq n$.

$\mathbf{f}(x_i)$= 22i -11 , 1≤ $i \leq n$.

$\mathbf{f}(y_i)$= 22 i -1 , 1≤ $i \leq n$.

$\mathbf{f}(x_{i'})$= 22i -11 , 1≤ $i \leq n$.

$\mathbf{f}(y_{i'})$= 22 i -1 , 1≤ $i \leq n$.

Then the edges are labeled as

$\mathbf{f}(u_i u_{i+1})$ =22i -5, 1≤ $i \leq n-1$.

$\mathbf{f}(u_i \boldsymbol{v_i})$ =22 i -20, 1≤ $i \leq n-1$.

$\mathbf{f}(\mathrm{u_i} \boldsymbol{v_{i'}})$ =22 i -19, 1≤ $i \leq n-1$.

$\mathbf{f}(v_i w_i)$ =22i -14, 1≤ $i \leq n-1$.

$\mathbf{f}(\boldsymbol{v_{i'}} w_{\boldsymbol{i'}})$ =22i-12, 1≤ $i \leq n-1$.

$\mathbf{f}(w_i u_{i+1})$ =22i -4, 1≤ $i \leq n-1$.

$\mathbf{f}(w_{i'} u_{i+1})$ =22i -3, 1≤ $i \leq n-1$.

$\mathbf{f}(u_i x_i)$ =22i-15, 1≤ $i \leq n-1$.

$\mathbf{f}(u_i x_{i'})$ =22i-17, 1≤ $i \leq n-1$.

$\mathbf{f}(x_{i'} y_{i'})$ =22i-8, 1≤ $i \leq n-1$.

$\mathbf{f}(x_i y_i)$ =22i-6, 1≤ $i \leq n-1$.

$\mathbf{f}(y_i u_{i+1})$ =22i, 1≤ $i \leq n-1$.

$\mathbf{f}(y_{i'} u_{i+1})$ =22i-2, 1≤ $i \leq n-1$.

Then we get distinct edge labels.

Hence F(Q_n) is Super Stolarsky-3 Mean graph.

Example 2.6: The SuperStolarsky-3 Mean labeling of F (Q_4) is given below.

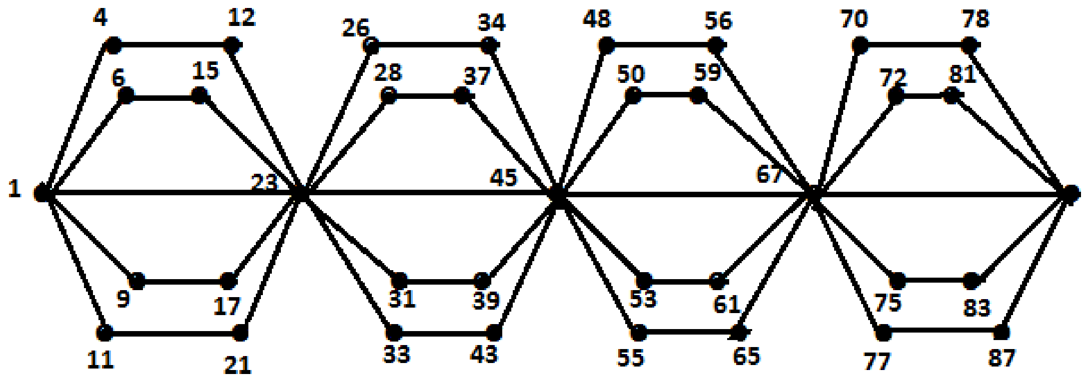

Figure 3

3. CONCLUSION

In this paper we discussed Super Stolarsky-3 Mean Labeling behavior of double, triple and Four Quadrilateral Snake graphs. The authors are of the opinion that the study of Super Stolarsky-3 Mean labeling of Quadrilateral Snake graphs shall be quite interesting and also will lead to new results.

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